Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$6C2 \times (2x)^4 \times \frac{1}{(4x^2)^2}$	**B1**	SOI **SC:** Condone errors in $(4^{-1})^2$ evaluation or interpretation for B1 only
	$15 \times 2^4 \times \frac{1}{4^2}$	**B1**	Identified as required term.
	15	**B1**	
		3	

Question	Answer	Marks	Guidance
2	Attempt to solve $f'(x) = 0$ or $f'(x) > 0$ or $f'(x) \geqslant 0$	**M1**	SOI
	$(x - 2)(x - 4)$	**A1**	2 and 4 seen
	(Least possible value of n is) 4	**A1**	Accept n = 4 or $n \geqslant 4$
		3	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx}=6x^2-10x-3$	**B1**	
	At $x = 2$, $\frac{dy}{dx}=24-20-3=1 \rightarrow a=1$	**M1** **A1**	
	$6=2+b \rightarrow b=4$	**B1FT**	Substitute $x = 2$, $y = 6$ in $y=(\textit{their } a)x+b$
	$6=16-20-6+c \rightarrow c=16$	**B1**	Substitute x =2, y = 6 into equation of curve
		5	

Question	Answer	Marks	Guidance
4(i)	Identifies common ratio as 1.1	**B1**	
	Use of $x(1.1)^{20}=20$	**M1**	SOI
	$x\left(=\frac{20}{(1.1)^{20}}\right)=3.0$	**A1**	Accept 2.97
		3	
4(ii)	$\textit{their}3.0\times\frac{\left[(1.1)^{21}-1\right]}{1.1-1}\rightarrow 192$	**M1** **A1**	Correct formula used for M mark. Allow 2.97 used from **(i)** Accept 190 from x = 2.97…
		2	

Question	Answer	Marks	Guidance
5(i)	$4\tan x + 3\cos x + \dfrac{1}{\cos x} = 0 \rightarrow 4\sin x + 3\cos^2 x + 1 = 0$	**M1**	Multiply by $\cos x$ or common denominator of $\cos x$
	$4\sin x + 3\left(1 - \sin^2 x\right) + 1 = 0 \rightarrow 3\sin^2 x - 4\sin x - 4 = 0$	**M1**	Use $\cos^2 x = 1 - \sin^2 x$ and simplify to 3-term quadratic in $\sin x$
	$\sin x = -\dfrac{2}{3}$	**A1**	AG
		3	
5(ii)	$2x - 20° = 221.8°, 318.2°$	**M1A1**	Attempt to solve $\sin(2x - 20) = -2/3$ (M1). At least 1 correct (A1)
	$x = 120.9°, 169.1°$	**A1** **A1FT**	FT for 290º – other solution. **SC** A1 both answers in radians
		4	

Question	Answer	Marks	Guidance
6	Equation of line is $y = mx - 2$	B1	**OR**
	$x^2 - 2x + 7 = mx - 2 \rightarrow x^2 - x(2+m) + 9 = 0$	M1	
	Apply $b^2 - 4ac(=0) \rightarrow (2+m)^2 - 4 \times 9\ (=0)$	*M1	
	$m = 4$ or -8	A1	
	$m = 4 \rightarrow x^2 - 6x + 9 = 0 \rightarrow x = 3$ $m = -8 \rightarrow x^2 + 6x + 9 = 0 \rightarrow x = -3$	DM1	
	(3, 10), (–3, 22)	A1A1	
	Alternative method for question 6		
	$\frac{dy}{dx} = 2x - 2$	B1	
	$2x - 2 = m$	M1	
	$x^2 - 2x + 7 = (2x-2)x - 2 = 2x^2 - 2x - 2$	M1	
	$x^2 - 9 = 0 \rightarrow x = \pm 3$	A1	
	(3, 10), (–3, 22)	A1A1	
	When $x = 3$, $m = 4$; when $x = -3$, $m = -8$	A1	
		7	

Question	Answer	Marks	Guidance
7(i)	Range of f is $0 < f(x) < 3$	B1B1	OE. Range cannot be defined using x
	Range of g is $g(x) > 2$	B1	OE
		3	
7(ii)	$\left(fg(x)=\right)\frac{3}{2\left(\frac{1}{x}+2\right)+1}=\frac{3x}{2+5x}$	B1B1	Second B mark implies first B mark
		2	
7(iii)	$y=\frac{3x}{2+5x} \rightarrow 2y+5xy=3x \rightarrow 3x-5xy=2y$	M1	Correct order of operations
	$x(3-5y)=2y \quad \rightarrow \quad x=\frac{2y}{3-5y}$	M1	Correct order of operations
	$\left((fg)^{-1}(x)\right)=\frac{2x}{3-5x}$	A1	
		3	

Question	Answer	Marks	Guidance
8(i)	$OA \times \frac{3}{8}\pi = 6$	**M1**	
	$OA = \frac{16}{\pi} = 5.093(0)$	**A1**	
8(ii)	$AB = their 5.0930 \times \tan\frac{3}{16}\pi$	**M1**	
	Perimeter $= 2 \times 3.4030 + 6 = 12.8$	**A1**	
8(iii)	Area $OABC$ = $(2 \times ½) \times their 5.0930 \times their 3.4030$	**M1**	
	Area sector = $½ \times (their 5.0930)^2 \times \frac{3}{8}\pi$	**M1**	
	Shaded area = $their 17.331 - their 15.279$ = 2.05	**M1A1**	

Question	Answer	Marks	Guidance
9(i)	$y = [(5x-1)^{1/2} \div \frac{3}{2} \ \div 5] \ [-2x]$	**B1** **B1**	
	$3 = \frac{27}{(3/2)\times 5} - 4 + c$	**M1**	Substitute $x = 2$, $y = 3$
	$c = 7 - \frac{18}{5} = \frac{17}{5} \to \left(y = \frac{2(5x-1)^{\frac{3}{2}}}{15} - 2x + \frac{17}{5} \right)$	**A1**	
9(ii)	$\mathrm{d}^2 y / \mathrm{d}x^2 = \left[½(5x-1)^{-1/2} \right] \ [\times 5]$	**B1** **B1**	
9(iii)	$(5x-1)^{1/2} - 2 = 0 \ \to \ 5x - 1 = 4$ $x = 1$	**M1A1**	Set $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and attempt solution (M1)
	$y = \frac{16}{25} - 2 + \frac{17}{5} = \frac{37}{15}$	**A1**	Or 2.47 or $\left(1, \frac{37}{15}\right)$
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^x} = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$ (> 0) hence minimum	**A1**	OE

Question	Answer	Marks	Guidance
10(i)	$\mathbf{AB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$, $\mathbf{BC} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$	**B1B1**	Condone reversal of labels
	AB.BC = 6 – 6 → = 0 (hence perpendicular)	**B1**	AG
10(ii)	$\mathbf{DC} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$	**B1**	Or: **CD** $= \begin{pmatrix} -2 \\ 4 \\ -6 \end{pmatrix}$
	AB = k**DC**	**M1**	OE Expect $k = \dfrac{3}{2}$ Or: **DC.BC** = 4 – 4 = 0 hence BC is also perpendicular to DC Or: **AB.DC** = 1 or **AB.CD** = –1, angle between lines is 0 or 180
	AB is parallel to DC, hence $ABCD$ is a trapezium	**A1**	
10(iii)	$\lvert\mathbf{AB}\rvert = \sqrt{9+36+81} = \sqrt{126} = 11.22$ $\lvert\mathbf{DC}\rvert = \sqrt{4+16+36} = \sqrt{56} = 7.483$ $\lvert\mathbf{BC}\rvert = \sqrt{4+1+0} = \sqrt{5} = 2.236$	**M1**	Method for finding at least 2 magnitudes
	Area = $\frac{1}{2}\left(theirAB + theirDC\right) \times theirBC = 20.92$	**M1A1**	OE

Question	Answer	Marks	Guidance
11(i)	$(y=)\ (x+2)^2-1$	**B1** **DB1**	2nd B1 dependent on 2 in bracket
	$x+2=(\pm)(y+1)^{1/2}$	**M1**	
	$x=-2+(y+1)^{1/2}$	**A1**	
11(ii)	$x^2=4+(y+1)-/+4(y+1)^{\frac{1}{2}}$	***M1A1**	SOI. Attempt to find x^2 .The last term can be – or + at this stage
	$(\pi)\int x^2\,(\mathrm{d}y)=(\pi)\left[5y+\dfrac{y^2}{2}-\dfrac{4(y+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$	**A2,1,0**	
	$(\pi)\left[15+\dfrac{9}{2}-\dfrac{64}{3}-\left(-5+\dfrac{1}{2}\right)\right]$	**DM1**	Apply y limits
	$\dfrac{8\pi}{3}$ or 8.38	**A1**	